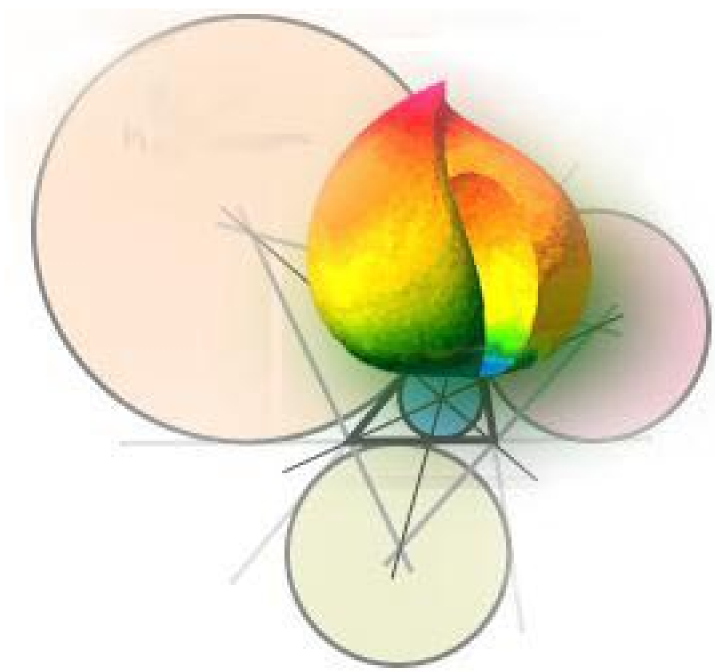
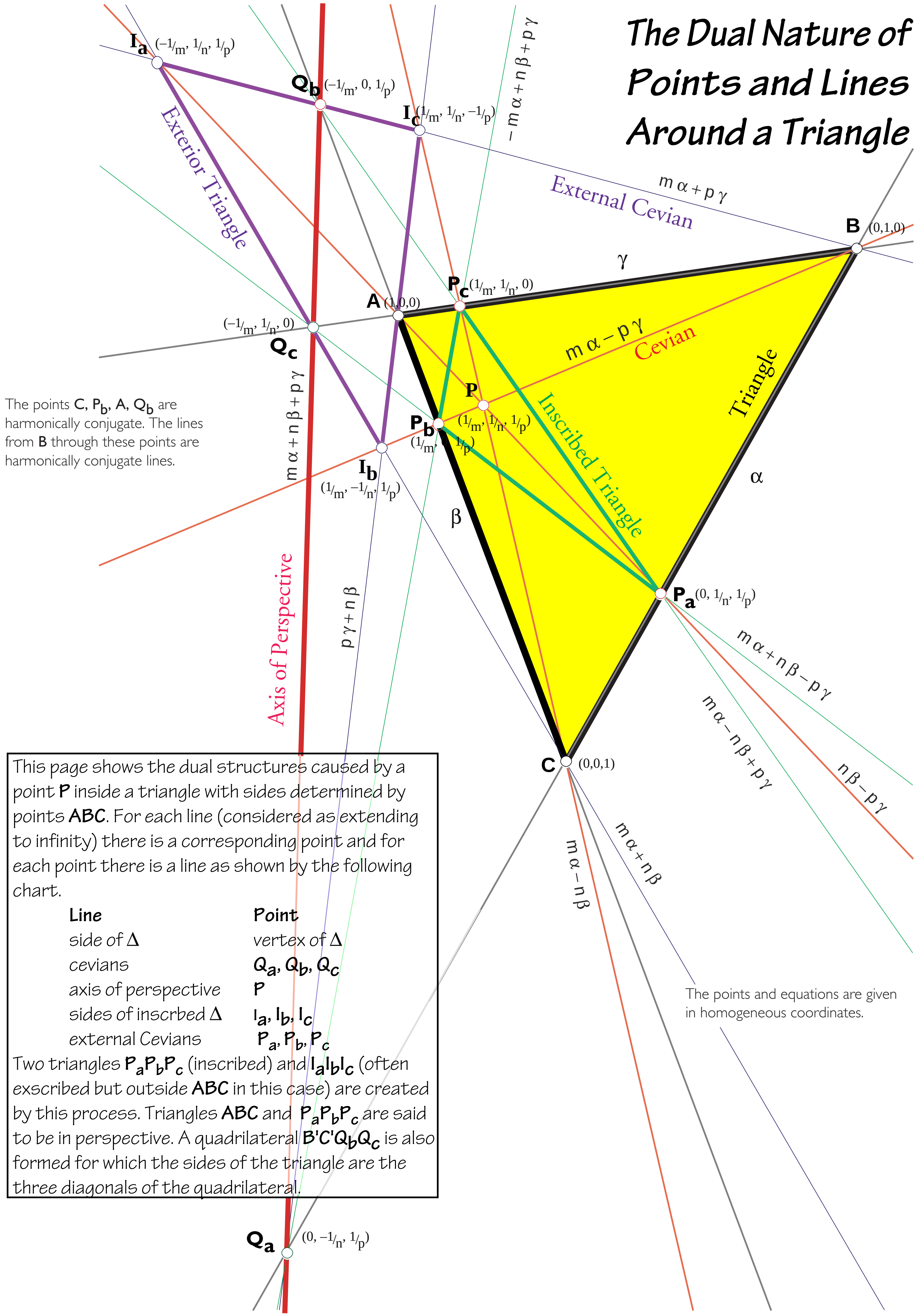
Geometry Pictures



Steve Sigur

The Dual Nature of Points and Lines Around a Triangle



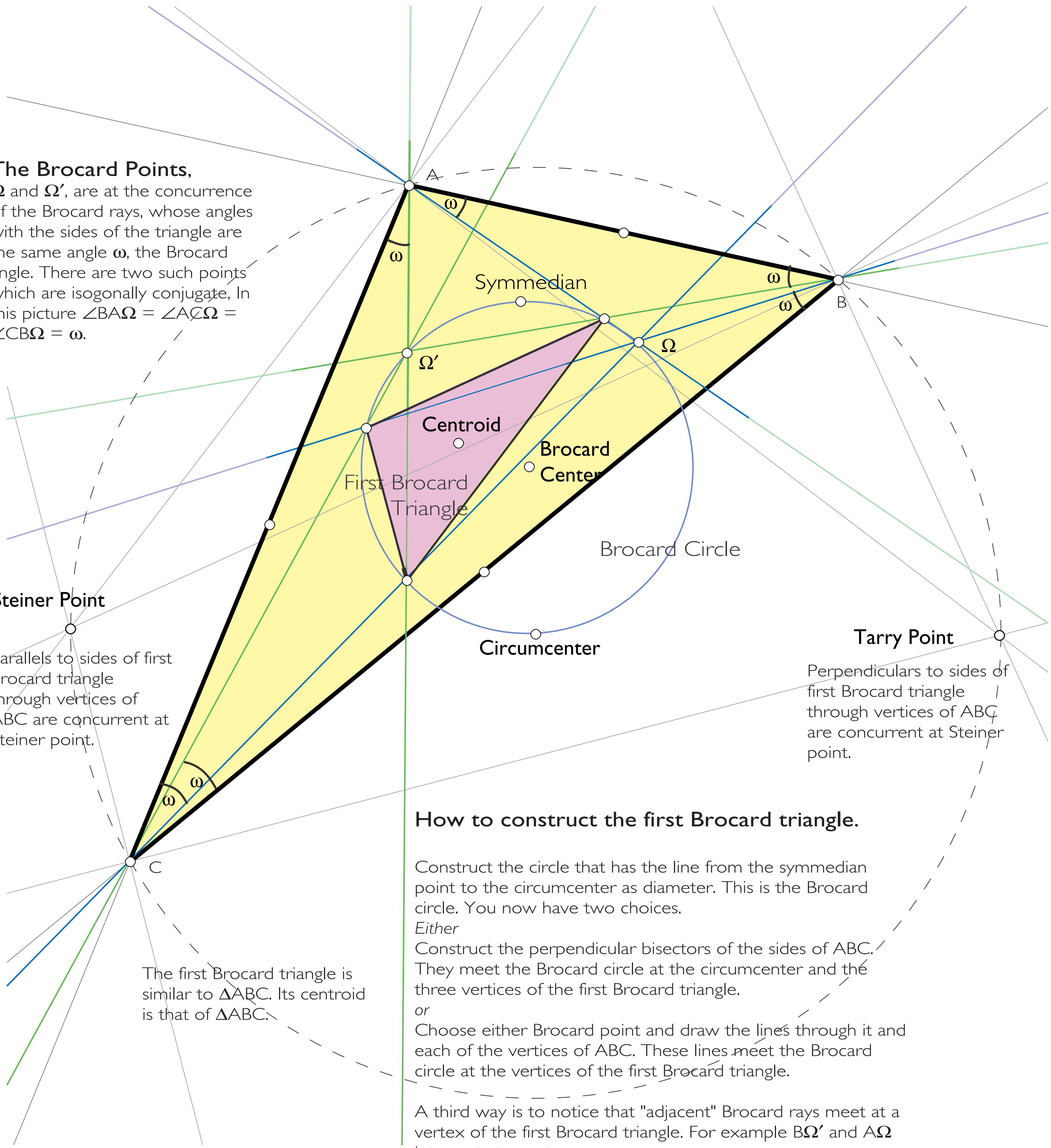
This page shows the dual structures caused by a point **P** inside a triangle with sides determined by points **ABC**. For each line (considered as extending to infinity) there is a corresponding point and for each point there is a line as shown by the following chart.

Line	**Point**
side of Δ	vertex of Δ
cevians	Q_a, Q_b, Q_c
axis of perspective	**P**
sides of inscrbed Δ	I_a, I_b, I_c
external Cevians	P_a, P_b, P_c

Two triangles $P_aP_bP_c$ (inscribed) and $I_aI_bI_c$ (often exscribed but outside **ABC** in this case) are created by this process. Triangles **ABC** and $P_aP_bP_c$ are said to be in perspective. A quadrilateral $B'C'Q_bQ_c$ is also formed for which the sides of the triangle are the three diagonals of the quadrilateral.

Brocard Geometry of the Triangle

Brocard circle, first Brocard triangle, Steiner and Tarry points



The Brocard Points,
Ω and **Ω′**, are at the concurrence of the Brocard rays, whose angles with the sides of the triangle are the same angle **ω**, the Brocard angle. There are two such points which are isogonally conjugate, In this picture ∠BA**Ω** = ∠AC**Ω** = ∠CB**Ω** = **ω**.

Steiner Point

Parallels to sides of first Brocard triangle through vertices of ABC are concurrent at Steiner point.

Tarry Point

Perpendiculars to sides of first Brocard triangle through vertices of ABC are concurrent at Steiner point.

The first Brocard triangle is similar to ΔABC. Its centroid is that of ΔABC.

How to construct the first Brocard triangle.

Construct the circle that has the line from the symmedian point to the circumcenter as diameter. This is the Brocard circle. You now have two choices.

Either

Construct the perpendicular bisectors of the sides of ABC. They meet the Brocard circle at the circumcenter and the three vertices of the first Brocard triangle.

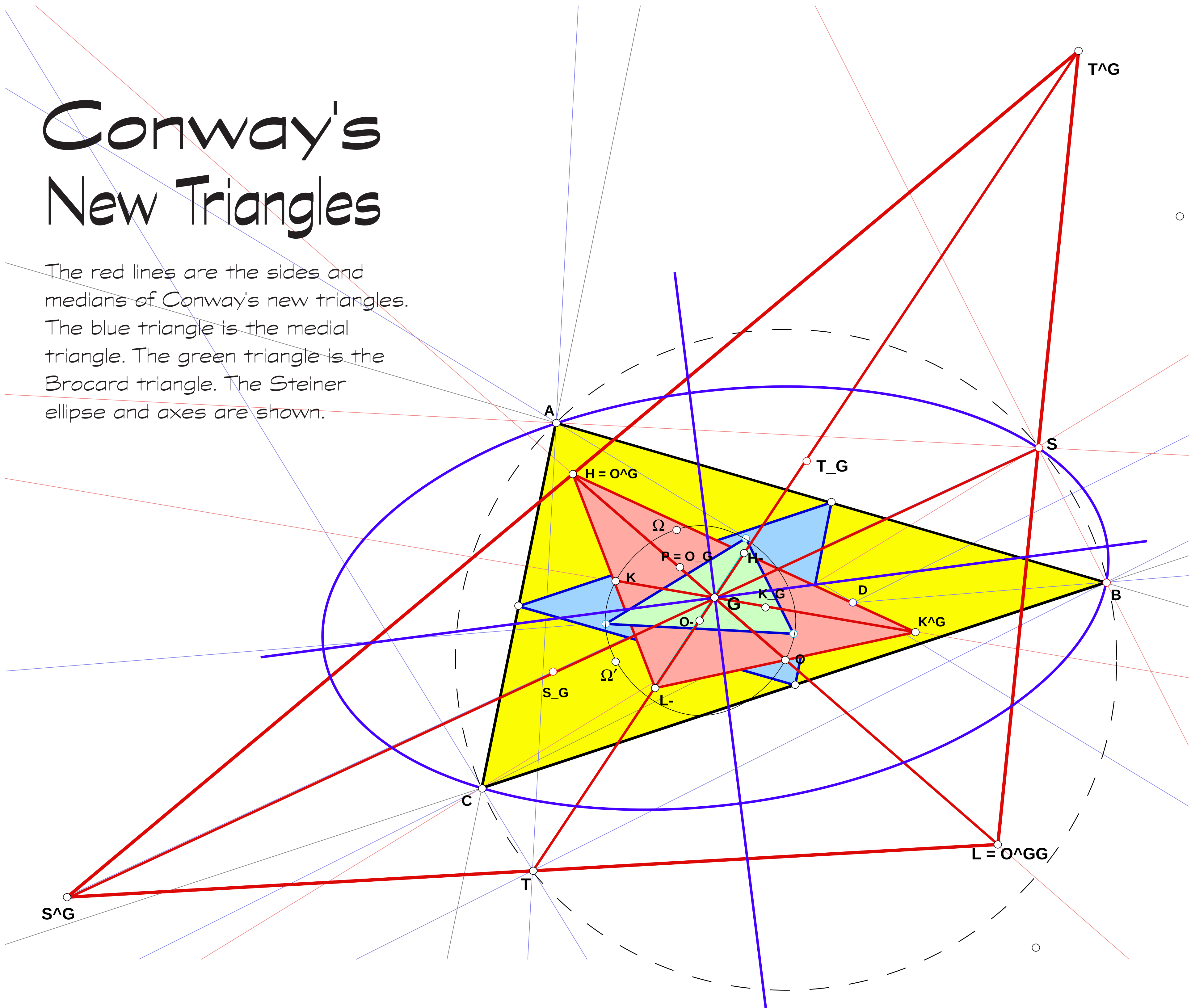
or

Choose either Brocard point and draw the lines through it and each of the vertices of ABC. These lines meet the Brocard circle at the vertices of the first Brocard triangle.

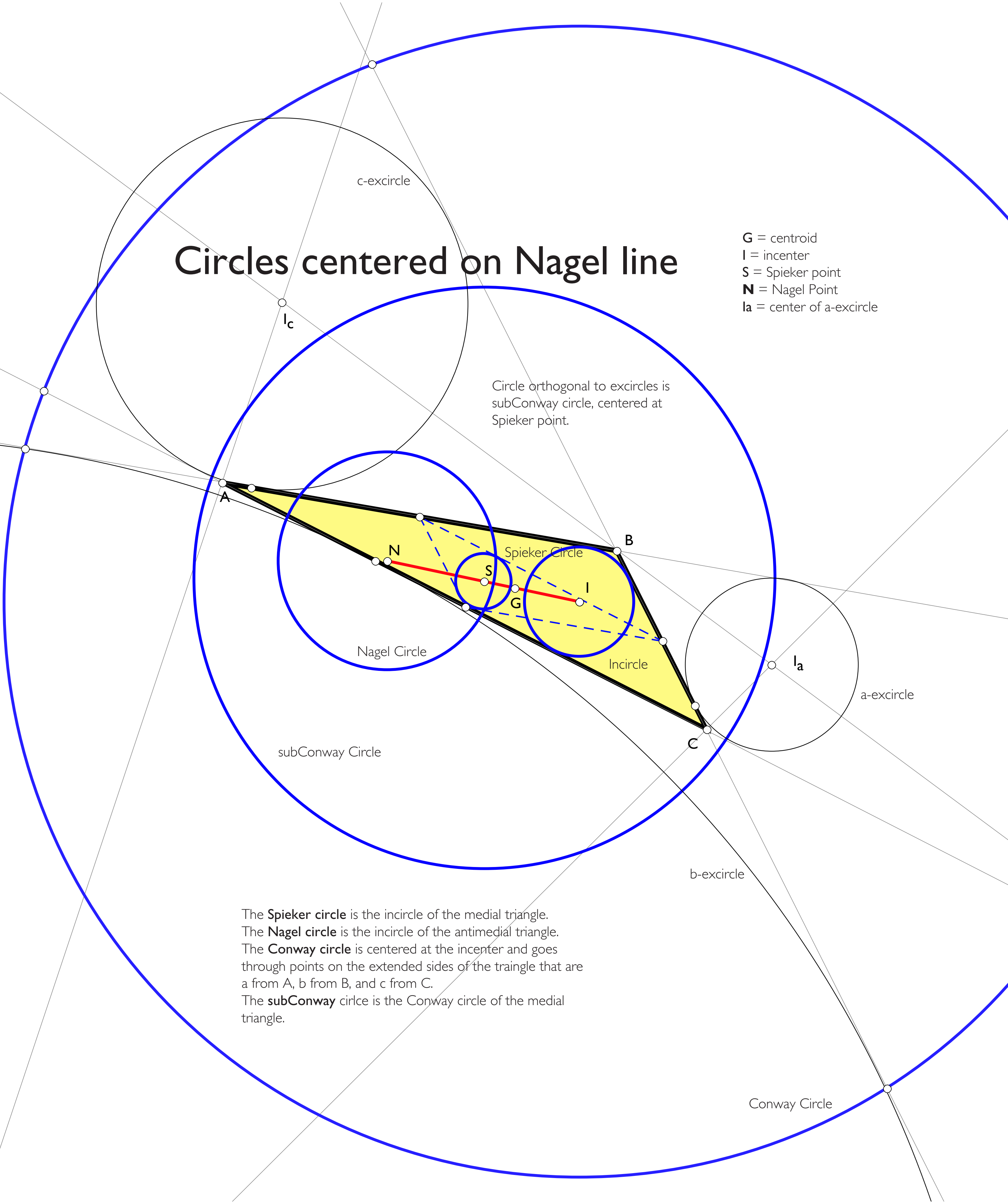
A third way is to notice that "adjacent" Brocard rays meet at a vertex of the first Brocard triangle. For example B**Ω′** and A**Ω** intersect at a vertex.

Conway's New Triangles

The red lines are the sides and medians of Conway's new triangles. The blue triangle is the medial triangle. The green triangle is the Brocard triangle. The Steiner ellipse and axes are shown.

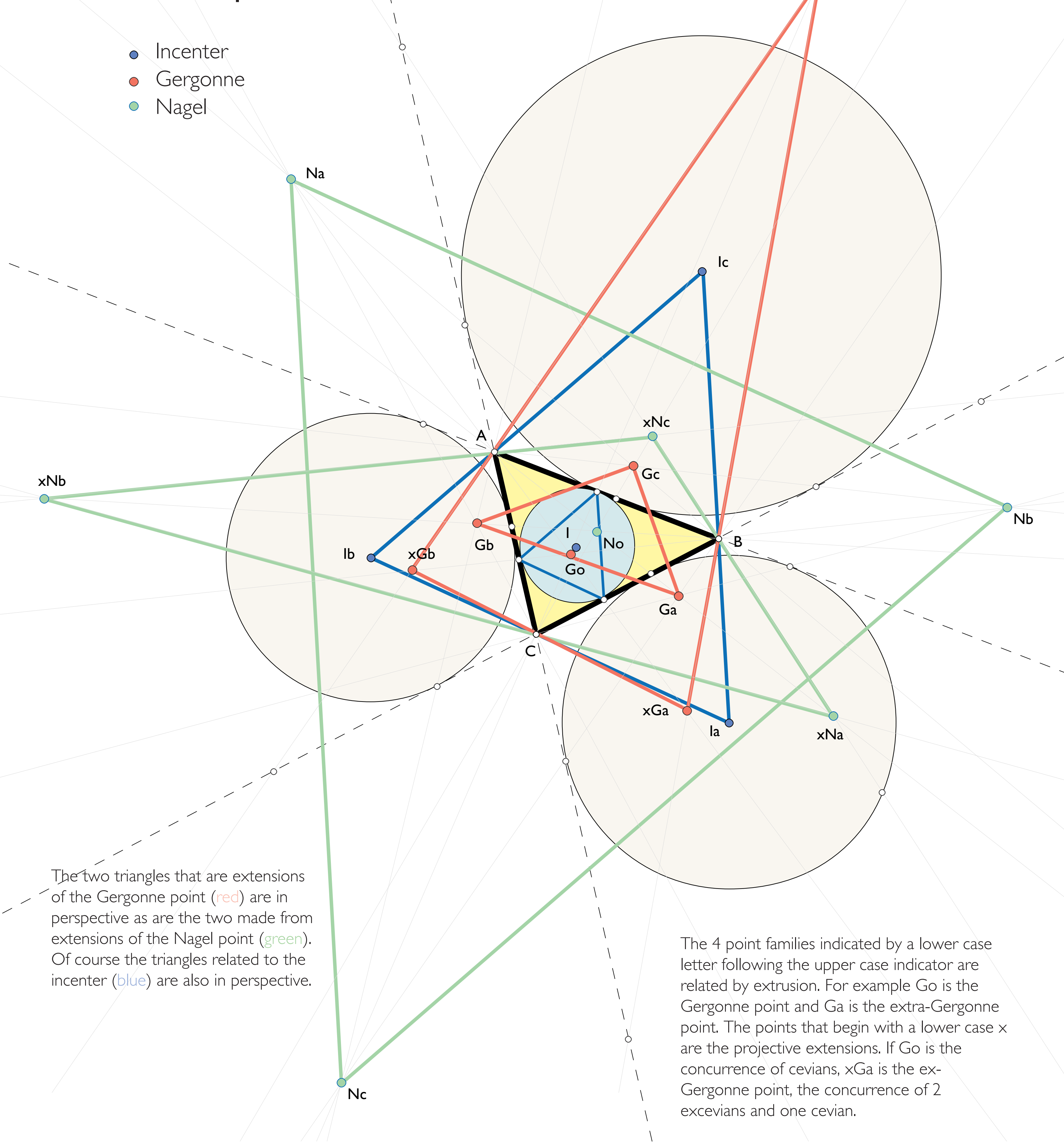


Circles centered on Nagel line



The **Spieker circle** is the incircle of the medial triangle.
The **Nagel circle** is the incircle of the antimedial triangle.
The **Conway circle** is centered at the incenter and goes through points on the extended sides of the traingle that are a from A, b from B, and c from C.
The **subConway** cirlce is the Conway circle of the medial triangle.

The Incenter, Gergonne, and Nagel families of points.



The two triangles that are extensions of the Gergonne point (red) are in perspective as are the two made from extensions of the Nagel point (green). Of course the triangles related to the incenter (blue) are also in perspective.

The 4 point families indicated by a lower case letter following the upper case indicator are related by extrusion. For example Go is the Gergonne point and Ga is the extra-Gergonne point. The points that begin with a lower case x are the projective extensions. If Go is the concurrence of cevians, xGa is the ex-Gergonne point, the concurrence of 2 excevians and one cevian.

ExtraNagel Lines, IsoSpieker points

The extraNagel triangle, ΔNaNbNc, is perspective to the antimedial triangle with the Nagel Point as perspector.
ΔIaIbIc is perspective to ΔNaNbNc with centroid as perspector.
Vertices of antimedial triangle are on extraNagel Δ.

The four isoSpieker points are colinear on the line connecting K, the symmedian points, and G, the centroid.

The extraSpieker circles are the excircles of the antimedial triangle.